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On R^2 corrections for 5D black holes

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Abstract: We study higher order corrections to the entropy of extremal BPS black holes/string in five dimensions. These corrections are due to supersymmetric completion of R^2 terms. These results can be used to make a connection between total bundle space of near horizon wrapped M2's and wrapped M5's in the presence of higher derivative terms. We also show how the corrected geometry removes the singularity of a small black hole.

Keywords: Black Holes in String Theory, Supergravity Models.

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1. Introduction

In recent years our understanding of corrections to the black hole entropy has increased considerably. In a gravitational theory, using the Wald entropy formula [1], one can find the contribution of higher order corrections to the tree level Bekenstein-Hawking area law formula. For extremal black holes in string theory taking into account the higher derivative terms, the corrected entropy has been evaluated in several papers including [2−4], where it has been shown that the results are in agreement with the entropy coming from the microstate counting in string theory.

An extremal black hole in four dimensions has $AdS_2 \times S^2$ near horizon geometry, while in five dimensions the near horizon geometry could be either $AdS_2 \times S^3$ or $AdS_3 \times S^2$. To compute the contribution of higher order corrections to a black hole with near horizon geometry $AdS_2 \times S^3$ one may simplify the Wald formula leading to the entropy function formalism [5]. While for those with $AdS_3 \times S^2$ near horizon geometry it is useful to work within the framework of the c-extremization [6]. The resulting higher derivative corrections to the Bekenstein-Hawking area law formula have to be compared with the microstate counting in string theory/M-theory.

The aim of this article is to study higher order corrections to the entropy of five dimensional $\mathcal{N} = 2$ BPS black holes. We note, however, that our understanding of the microscopic origin of the entropy for these black holes is quite limited. Actually we note that although the microscopic origin of the entropy for $\mathcal{N} = 8, 4$ five dimensional (rotating) black holes has been understood for a decade [7, 8], for the case of $\mathcal{N} = 2$ black holes it has not been fully understood yet (see however [9]). A new attempt has recently been made to understand the microscopic origin of the entropy for $\mathcal{N} = 2$ black holes in five dimensions. More precisely the microscopic counting of the five dimensional rotating black hole arising from wrapped M2-branes in Calabi-Yau compactification of M-theory has been studied in [10] where the authors established a connection between this black hole and another well understood black hole by making use of an embedding of space-time in the total space of the U(1) gauge bundle over near horizon geometry of the black holes. The work of [10] was limited to the case of near zero-entropy, zero-temperature and maximally rotating black hole.

To study higher order corrections to $\mathcal{N}=2$ five dimensional BPS black holes we will work with the full 5D supersymmetry invariant four-derivative action, corresponding to the supersymmetric completion of the four-derivative Chern-Simons term which has recently been obtained in [11].

The article is organized as follows. In section 2 we will fix our notation where we present the five dimensional action obtained in [11]. In section 3 we will study higher order corrections to the entropy of a five dimensional extremal BPS black hole using the fully supersymmetrized higher derivative terms. We then compare the result with the corrections coming from the bosonic Gauss-Bonnet action. We will also see how the higher derivative terms remove the singularity of the small black hole. In section 4 we will first review the five dimensional black string solution in the presence of R^2 terms; then using the corrected near horizon geometry of the black string solution we will extend the considerations of [10] to the case including R^2 terms. The last section is devoted to discussions and conclusions.

2. Basic setup

In this section we present the result of [11] to fix our notation. To study $\mathcal{N}=2$ supergravity in five dimensions in the presence of R^2 corrections, the authors of [11] utilize the superconformal formalism [12]. This approach, in particular, is useful when we want to write the explicit form of the action. In this approach we start with a five dimensional theory which is invariant under a larger group, i.e. superconformal group, and construct a conformal supergravity. Then by imposing a gauge fixing condition the conformal supergravity is reduced to the standard supergravity model.

The representation of superconformal group includes Weyl, vector and hyper multiplets. The bosonic part of the Weyl multiplet contains the vielbein e_μ^a, two-form auxiliary field v_{ab}, and a scalar auxiliary field D. The bosonic part of the vector multiplet contains one-form gauge field A^I and scalar fields X^I, where $I=1,\cdots,n_v$ labels generators of a gauge group. The hyper multiplet contains scalar fields $\mathcal{A}_\alpha^i$ where $i=1,2$ is SU(2) doublet index and $\alpha=1,\cdots,2r$ refers to $USp(2r)$ group. Although we won't couple the theory to matters, we shall consider the hyper multiplet to gauge fix the dilatational symmetry which reduces the action to the standard $\mathcal{N}=2$ supergravity action.

In this notation at leading order the bosonic part of the action is [11]

$$I=\frac{1}{16\pi G_5}\int d^5x\mathcal{L}_0, \tag{2.1}$$

with

$$\begin{aligned}\mathcal{L}_0 =\ &\partial_a\mathcal{A}_\alpha^i\partial^a\mathcal{A}_i^\alpha+(2\nu+\mathcal{A}^2)\frac{D}{4}+(2\nu-3\mathcal{A}^2)\frac{R}{8}+(6\nu-\mathcal{A}^2)\frac{v^2}{2}+2\nu_I F_{ab}^I v^{ab}\\ &+\frac{1}{4}\nu_{IJ}(F_{ab}^I F^{J\,ab}+2\partial_a X^I\partial^a X^J)+\frac{g^{-1}}{24}C_{IJK}\epsilon^{abcde}A_a^I F_{bc}^J F_{de}^K,\end{aligned} \tag{2.2}$$

where $\mathcal{A}^2 = \mathcal{A}^i_{\alpha\ ab}\mathcal{A}^{\alpha\ ab}_i$, $v^2 = v_{ab}v^{ab}$ and

$$\nu = \frac{1}{6}C_{IJK}X^IX^JX^K, \qquad \nu_I = \frac{1}{2}C_{IJK}X^JX^K, \qquad \nu_{IJ} = C_{IJK}X^K. \tag{2.3}$$

To fix the gauge it is convenient to set $\mathcal{A}^2 = -2$. Then integrating out the auxiliary fields by making use of their equations of motion one finds

$$\mathcal{L}_0 = R - \frac{1}{2}G_{IJ}F^I_{ab}F^{Jab} - \mathcal{G}_{ij}\partial_a\phi^i\partial^a\phi^j + \frac{g^{-1}}{24}\epsilon^{abcde}C_{IJK}F^I_{ab}F^J_{cd}A^K_e. \tag{2.4}$$

The parameters in the action (2.4) are defined by

$$G_{IJ} = -\frac{1}{2}\partial_I\partial_J\log\nu|_{\nu=1}\ , \qquad \mathcal{G}_{ij} = G_{IJ}\ \partial_iX^I\partial_jX^J|_{\nu=1}\ , \tag{2.5}$$

where ∂_i refers to a partial derivative with respect to the scalar fields ϕ^i. In fact doing this, we recover the very special geometry underlies the theory in the leading order.

We can also find higher derivative terms in the action using the superconformal language. Actually, the supersymmetrized higher order action with four-derivative terms has recently been obtained in [11]. The corresponding action is

$$\begin{aligned}\mathcal{L}_1 = \frac{c_{2I}}{24}\Big(&\frac{1}{16}g^{-1}\epsilon_{abcde}A^{Ia}C^{bcfg}C^{de}{}_{fg} + \frac{1}{8}X^IC^{abcd}C_{abcd} + \frac{1}{12}X^ID^2 + \frac{1}{6}F^{Iab}v_{ab}D\\ &-\frac{1}{3}X^IC_{abcd}v^{ab}v^{cd} - \frac{1}{2}F^{Iab}C_{abcd}v^{cd} + \frac{8}{3}X^Iv_{ab}\hat{\mathcal{D}}^b\hat{\mathcal{D}}_cv^{ac}\\ &+\frac{4}{3}X^I\hat{\mathcal{D}}^av^{bc}\hat{\mathcal{D}}_av_{bc} + \frac{4}{3}X^I\hat{\mathcal{D}}^av^{bc}\hat{\mathcal{D}}_bv_{ca} - \frac{2}{3}e^{-1}X^I\epsilon_{abcde}v^{ab}v^{cd}\hat{\mathcal{D}}_fv^{ef}\\ &+\frac{2}{3}e^{-1}F^{Iab}\epsilon_{abcde}v^{cd}\hat{\mathcal{D}}_fv^{ef} + e^{-1}F^{Iab}\epsilon_{abcde}v^c{}_f\hat{\mathcal{D}}^dv^{ef}\\ &-\frac{4}{3}F^{Iab}v_{ac}v^{cd}v_{db} - \frac{1}{3}F^{Iab}v_{ab}v^2 + 4X^Iv_{ab}v^{bc}v_{cd}v^{da} - X^I(v_{ab}v^{ab})^2\Big),\end{aligned} \tag{2.6}$$

where C_{abcd} is the Weyl tensor defined as

$$C^{ab}{}_{cd} = R^{ab}{}_{cd} + \frac{1}{6}R\delta^{[a}{}_{[c}\delta^{b]}{}_{d]} - \frac{4}{3}\delta^{[a}{}_{[c}R^{b]}{}_{d]}\ . \tag{2.7}$$

The double covariant derivative of v_{ab} has curvature contributions given by

$$v_{ab}\hat{\mathcal{D}}^b\hat{\mathcal{D}}_cv^{ac} = v_{ab}\mathcal{D}^b\mathcal{D}_cv^{ac} + \frac{2}{3}v^{ac}v_{cb}R_a{}^b + \frac{1}{12}v_{ab}v^{ab}R\ . \tag{2.8}$$

In general it is quite difficult to solve the equations of motion coming from the action which contains both $\mathcal{L}_0$ and $\mathcal{L}_1$. Nevertheless since we are looking for supersymmetric solutions, we will use the supersymmetry transformations to simplify the equations. The supersymmetry variations of the fermions in Weyl, vector and hyper multiplets (taking only the bosonic terms) are[1] (see e.g. [12])

$$\begin{aligned}
\delta\psi^i_\mu &= \mathcal{D}_\mu\varepsilon^i + \frac{1}{2}v^{ab}\gamma_{\mu ab}\varepsilon^i - \gamma_\mu\eta^i, \\
\delta\chi^i &= D\varepsilon^i - 2\gamma^c\gamma^{ab}\varepsilon^i\hat{\mathcal{D}}_a v_{bc} + \gamma\cdot\hat{R}(V)^i_j\varepsilon^j - 2\gamma^a\varepsilon^i\epsilon_{abcde}v^{bc}v^{de} + 4\gamma\cdot v\eta^i, \\
\delta\Omega^{Ii} &= -\frac{1}{4}\gamma\cdot F^I\varepsilon^i - \frac{1}{2}\gamma^a\partial_a X^I\varepsilon^i - X^I\eta^i, \\
\delta\zeta^\alpha &= \gamma^a\partial_a\mathcal{A}^\alpha_i\varepsilon^i - \gamma\cdot v\varepsilon^i\mathcal{A}^\alpha_i + 3\mathcal{A}^\alpha_i\eta^i,
\end{aligned} \tag{2.9}$$

where garavitino ψ^i_μ and the auxiliary Majorana spinor χ^i come from the Weyl multiple, while the gaugino Ω^{Ii} and ζ^α come from vector and hyper multiplets, respectively.

3. Black hole solution

In this section we consider a five dimensional extremal BPS black hole[2] whose near horizon geometry is $AdS_2 \times S^3$. When we are dealing with an extremal black hole with AdS_2 near horizon geometry, it is more appropriate to work with entropy function formalism [5]. In fact this approach has been used to study five dimensional extremal black holes in Heterotic string theory in the presence of the higher derivative terms given by Gauss-Bonnet action [13].[3] It has been shown that this *bosonic* term is enough to correctly reproduce the microscopic entropy coming from microstate counting in string theory. It is the aim of this section to study the five dimensional extremal BPS black hole in the presence of higher derivative terms which come from supersymmetrized action. We will also compare the result with the case where only bosonic Gauss-Bonnet term is present. In fact we are following [18] where the entropy of four dimensional extremal BPS black holes have been calculated using supersymmetrized action with help of entropy function formalism.

Let us start with the following ansatz for the near horizon geometry

$$ds^2 = l_A^2 ds^2_{ADS_2} + l_S^2 ds^2_{S^3}, \qquad X^I = cont. \qquad F^I_{rt} = e^I, \qquad v_{rt} = V. \tag{3.1}$$

Then the entropy function is given by $\mathcal{E} = 2\pi(e^I q_I - f_0 + f_1)$, where f_0 is the leading order contribution coming from quadratic part of the action, which is

$$f_0 = \frac{1}{2}l_A^2 l_S^3\left[\frac{\nu-1}{2}D + \frac{\nu+3}{2}\left(\frac{3}{l_S^2} - \frac{1}{l_A^2}\right) - \frac{2(3\nu+1)}{l_A^4}V^2 - \frac{4\nu_I e^I}{l_A^4}V - \frac{\nu_{IJ}e^I e^J}{2l_A^4}\right], \tag{3.2}$$

and the higher order contribution, f_1, comes from the four-derivative terms which for our ansatz is

$$\begin{aligned}
f_1 = \frac{c_{2I}}{48}l_A^2 l_S^3\Bigg[&\frac{X^I}{4}\left(\frac{1}{l_S^2} - \frac{1}{l_A^2}\right)^2 + \frac{4V^4}{l_A^8}X^I + \frac{4V^3}{3l_A^8}e^I - \frac{DV}{3l_A^4}e^I + \frac{D^2}{12}X^I \\
&-\frac{2V^2X^I}{3l_A^4}\left(\frac{3}{l_S^2} + \frac{5}{l_A^2}\right) - \frac{Ve^I}{l_A^4}\left(\frac{1}{l_A^2} - \frac{1}{l_S^2}\right)\Bigg].
\end{aligned} \tag{3.3}$$

[1]Here the covariant curvature $\hat{R}^{ij}_{\mu\nu}(V)$ is defined by $\hat{R}^{ij}_{\mu\nu}(V) = 2\partial_{[\mu}V^{ij}_{\nu]} - 2V^i_{[\mu k}V^{kj}_{\nu]}$ + fermionic terms, where V^{ij}_μ is a boson in the Weyl multiplet which is in **3** of the SU(2). We note, however, that for the ansatz we are going to consider, this term vanishes.

[2]Explicit solutions of $\mathcal{N} = 2$ 5D supersymmetric black holes can be found for example in [14].

[3]Higher order corrections to 5D BH have also been studied in [15 – 17].

At leading order where only f_0 contributes, one may integrate out the the auxiliary fields by extremizing the entropy function with respect to them. Doing so, we arrive at

$$f_0 = \frac{1}{2}l_A^2 l_S^3 \left[\frac{6}{l_S^2} - \frac{2}{l_A^2} + \frac{1}{2l_A^4}(\nu_I\nu_J - \nu_{IJ})e^I e^J\right] = \frac{1}{2}l_A^2 l_S^3 \left(\frac{6}{l_S^2} - \frac{2}{l_A^2} + \frac{G_{IJ}e^I e^J}{l_A^4}\right). \tag{3.4}$$

It is then easy to extremize the entropy function with respect to the parameters to find l_A, l_S, X^I and e^I. In particular for STU model, where $C_{123} = 1$ and the other components are zero, one gets

$$l_S = 2l_A = (q_1q_2q_3)^{1/6}, \quad X^I = \frac{(q_1q_2q_3)^{1/3}}{q_I}, \quad e^I = \frac{1}{2}\frac{(q_1q_2q_3)^{1/2}}{q_I}, \tag{3.5}$$

and the corresponding black hole entropy is $S = 2\pi\sqrt{q_1q_2q_3}$.

In general it is difficult to do the same while the higher order corrections are also taken into account. Nevertheless as far as a BPS solution is concerned we can use the supersymmetry transformations (2.9) to simplify the equations. In fact in this case we do not even need to solve the equations. Actually for the ansatz we are considering, setting the supersymmetry transformation to zero, one may write D, V, X^I and l_s in terms of l_A. The remaining parameter, l_A, can then be found from equation of motion of the auxiliary field D. Of course it is easy to see that the obtained solution is indeed a solution of the equations of motion.

More explicitly, for a BPS solution setting the supersymmetry transformation to zero for the ansatz (3.1) we find

$$D = -\frac{3}{l_A^2}, \qquad e^I = -\frac{4}{3}VX^I, \qquad V = -\frac{3}{4}l_A, \qquad l_S = 2l_A. \tag{3.6}$$

From these relations one may set $X^I = \frac{e^I}{l_A}$. On the other hand defining $E = \frac{1}{6}C_{IJK}e^I e^J e^K$ we have

$$\nu = \frac{1}{l_A^3}E, \qquad \nu_I e^I = \frac{3}{l_A^2}E, \qquad \nu_{IJ}e^I e^J = \frac{6}{l_A}E. \tag{3.7}$$

Using this notation, the equation of motion for the auxiliary field D reads

$$E - l_A^3 + \frac{l_A^3}{12}c_{2I}\left(\frac{DX^I}{6} - \frac{Ve^I}{3l_A^4}\right) = 0 \tag{3.8}$$

so that $l_A = \frac{1}{2}(8E - \frac{c_{2I}e^I}{6})^{1/3}$. By making use of the expressions for the parameters D, V, X^I and l_S, the entropy function gets the following simple form

$$\mathcal{E} = 2\pi(q_I e^I - 4E + \frac{1}{8}c_{2I}e^I) = 2\pi\left[\left(q_I + \frac{1}{8}c_{2I}\right)e^I - \frac{2}{3}C_{IJK}e^I e^J e^K\right] \tag{3.9}$$

Extremizing the entropy function with respect to e^I we get

$$2C_{IJK}e^J e^K = q_I + \frac{1}{8}c_{2I} \tag{3.10}$$

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which in principle can be solved to find e^I in terms of the charges, c_{2I} and parameters C_{IJK}'s. The entropy is also given by $S = \frac{4\pi}{3} q_I^+ e^I$.[4] In particular for STU model we get $e^I = (q_1^+ q_2^+ q_3^+)^{1/2}/2q_I^+$. Plugging this into the expressions of l_A and X^I, we obtain

$$2l_A = (q_1^+ q_2^+ q_3^+)^{1/6}\left(1 - \frac{c_{2I}}{12}\frac{1}{q_I^+}\right)^{1/3}, \qquad X^I = \frac{(q_1^+ q_2^+ q_3^+)^{1/3}}{q_I^+}\left(1 - \frac{c_{2I}}{12}\frac{1}{q_I^+}\right)^{-1/3}. \tag{3.11}$$

Finally the entropy is found to be

$$S = 2\pi\sqrt{q_1^+ q_2^+ q_3^+}. \tag{3.12}$$

It is very interesting in the sense that the entropy of the black hole at R^2 level has the same form as the tree level one, except that the charges q_I's are replaced by shifted charges q_I^+'s.

This result can be used to see how the higher order corrections stretch the horizon. For example if we start from a classical solution with $q_1 = 0$ we get vanishing horizon (small black hole) and vanishing entropy. But adding the R^2 terms we get a smooth solution with a non-zero entropy give by

$$S = \pi\sqrt{\frac{c_{21}}{2} q_2^+ q_3^+}. \tag{3.13}$$

This procedure could also be used to understand, upon dimensional reduction to four dimensions, the single-charge small black hole studied in [19].

We note that in the above considerations we have used the supersymmetry transformations to simplify the computations and therefore the solution is supersymmetric. Thus it does not exclude the existence of other solutions. In fact we would expect to have another solution corresponding to a non-BPS solution. Actually one could start from a more general constraint than (3.6) as follows

$$D = \frac{\alpha}{l_A^2}, \qquad X^I = \beta\frac{e^I}{l_A}, \qquad V = \gamma l_A, \qquad l_S = 2l_A \tag{3.14}$$

and solve the equations for parameters (α, β, γ). Doing so, for fixed l_A given above, we find two solutions: $(-3, 1, -3/4)$ and $(3, -1, -3/4)$. The first one is the solution we have studied, but the second one which is not supersymmetric leads to the following entropy function

$$\mathcal{E} = 2\pi\left[\left(q_I - \frac{3}{8}c_{2I}\right)e^I - \frac{2}{3}C_{IJK}e^I e^J e^K\right]. \tag{3.15}$$

It is straightforward to extremize the entropy function with respect to e^I's to get the parameters in terms of q_I's, though we won't do that here.

It is also instructive to compare the results with the case where the higher order corrections are given in terms of the Gauss-Bonnet action. It is known that this term cannot be supersymmetrized, still it is interesting to see what would be the corresponding corrections. In our notation the Gauss-Bonnet term is given by

$$\mathcal{L}_{GB} = \frac{c_{2I}X^I}{2^8 \cdot 3\pi^2}\left(R^{abcd}R_{abcd} - 4R^{ab}R_{ab} + R^2\right). \tag{3.16}$$

[4] We use a notation in which $q_I^+ = q_I + \frac{1}{8}c_{2I}$.

It can be shown that with the specific coefficient we have chosen for the Gauss-Bonnet action, the corresponding entropy function, fixing the auxiliary fields as (3.6) for the tree level action, is the same as (3.9) and therefore we get the same results as those in supersymmetrized action. This might be understood from the fact that the attractor mechanism [20] works for both supersymmetric and non-supersymmetric cases [21]. In fact it is believed that the important role is playing by the extremality (the AdS_2 near horizon geometry) rather than the supersymmetry [22].

4. Black string and rotating black hole

In this section we first review the results of [23] where the authors have studied the black string solution in the presence of higher derivative terms given by (2.6). Then by making use of the corrected near horizon geometry of the black string solution we will extend the considerations of [10] to find the corrected near horizon geometry of the five dimensional rotating black hole. On the other hand due to the attractor mechanism the near horizon geometry is sufficient for finding the entropy of the corresponding black hole/string [5]. Therefore following [10] one would expect that this relation could be useful to understand the microscopic origin of some black hole/string using the known cases in the presence of R^2 corrections.

Consider a five dimensional extremal BPS black string solution whose near horizon geometry is $AdS_3 \times S^2$. Using the isometry of the near horizon geometry the most general ansatz for the near horizon solution is

$$ds^2 = l_A^2 ds^2_{ADS_3} + l_S^2 ds^2_{S^2}, \quad X^I = cont. \quad F^I_{\theta\phi} = \frac{p^I}{2}\sin\theta, \quad v_{\theta\phi} = V\sin\theta, \tag{4.1}$$

with constant D. By making use of the c-extremization [6] we can fix the parameters of the ansatz as follows. In this method we first define c-function whose critical points correspond to the solutions of the equations of motion. Then evaluating the c-function at critical points gives the average of the left and right moving central charges of the associated dual CFT [6].

In five dimensions the c-function is given by

$$c = -6\, l_A^3 l_S^2\, \mathcal{L}, \tag{4.2}$$

where $\mathcal{L}$ is the Lagrangian evaluated on the above ansatz. The parameters of the ansatz are obtained by extremizing this function with respect to them. In leading order when the five dimensional action is given by (2.4) the above equations can be solved leading to

$$l_A = 2l_S = \left(\frac{1}{6}C_{IJK}p^Ip^Jp^K\right)^{1/3}, \qquad X^I = \frac{p^I}{\left(\frac{1}{6}C_{IJK}p^Ip^Jp^K\right)^{1/3}}. \tag{4.3}$$

Plugging these into (4.2), we find the central charge for the black string at two-derivative level as $c = C_{IJK}p^Ip^Jp^K$.

In the presence of higher derivative terms given by (2.6), it is in general difficult to solve the equations coming from extremization of the c-function. Nevertheless as far

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as a BPS solution is concerned we can use the supersymmetry transformations (2.9) to simplify the equations. In fact for the ansatz we are considering, setting the supersymmetry transformation to zero, we will be able to write the parameters D, V, X^I and l_s in terms of l_A. Then the remaining parameter, l_A, can be found from equation of motion of the auxiliary field D. More explicitly, from the supersymmetry transformations (2.9) for the ansatz (4.1) we find [23]

$$D = \frac{12}{l_S^2}, \qquad p^I = -\frac{8}{3}VX^I, \qquad V = -\frac{3}{8}l_A, \qquad l_A = 2l_S. \tag{4.4}$$

On the other hand the equation of motion of the auxiliary field D is

$$\frac{1}{6}C_{IJK}X^IX^JX^K + \frac{c_{2I}}{72}\left(DX^I + \frac{Vp^I}{l_S^4}\right) = 1, \tag{4.5}$$

which by making use of (4.4) leads to $l_A^3 = \frac{1}{6}C_{IJK}p^Ip^Jp^K + \frac{1}{12}c_{2I}p^I$. Finally the corrected central charge of the associated dual CFT is given by

$$c = C_{IJK}p^Ip^Jp^K + \frac{3}{4}c_{2I}p^I. \tag{4.6}$$

The equation of motion of the auxiliary field D may also be recast into the following form

$$\frac{1}{6}C_{IJK}X^IX^JX^K + \frac{1}{12l_A^2}c_{2I}X^I = 1. \tag{4.7}$$

We note that, setting $c_{2I} = 0$, it reduces to $\nu = 1$ where one can recover very special geometry underlies the theory at leading order. Therefore we would like to interpret this expression as a generalization of $\nu = 1$ when the R^2 corrections are also taken into account. This is analogous to the one-loop correction to the prepotential of four dimensional $\mathcal{N} = 2$ supergravity which it is given by $F = \frac{1}{6}\frac{C_{IJK}X^IX^JX^K}{X^0} + \Lambda^2\,\frac{c_{2I}X^I}{X^0}$.

It is then natural to define the dual coordinates X_I as

$$X_I = \frac{1}{6}C_{IJK}X^JX^K + \frac{1}{12l_A^2}c_{2I}, \tag{4.8}$$

such that $X_IX^I = 1$. From the supersymmetry conditions (4.4) one may set $X^I = \frac{p^I}{l_A}$ to find

$$X^I = \frac{p^I}{(\frac{1}{6}C_{IJK}p^Ip^Ip^K + \frac{1}{12}c_{2I}p^I)^{1/3}}, \qquad X_I = \frac{\frac{1}{6}C_{IJK}p^Ip^K + \frac{1}{12}c_{2I}}{(\frac{1}{6}C_{IJK}p^Ip^Ip^K + \frac{1}{12}c_{2I}p^I)^{2/3}}. \tag{4.9}$$

The magnetic central charge determining the tension of the string which is a function of the moduli is defined by $Z_m = X_Ip^I$. Using the above expression for X_I we get

$$Z_m = \left(\frac{1}{6}C_{IJK}p^Ip^Jp^K + \frac{1}{12}c_{2I}p^I\right)^{1/3}. \tag{4.10}$$

Now we would like to extend the considerations of [10] to study corrections to the near horizon geometry of the rotating extremal BPS black hole. To do this we start from the corrected near horizon geometry of the extremal black string solution which can be read from (4.1) as follows

$$
\begin{aligned}
ds^2 &= \frac{l_A^2}{4}\left(dx^2 - 2rdxdr + \frac{dr^2}{r^2} + d\theta^2 + \sin^2\theta\, d\phi^2\right), \qquad X^I = \frac{p^I}{l_A}, \\
F^I_{\theta\phi} &= \frac{p^I}{2}\sin\theta\,, \qquad\qquad\qquad\qquad\qquad\qquad\qquad v = -\frac{3}{8}l_A\sin\theta,
\end{aligned}
\tag{4.11}
$$

where $l_A = \left(\frac{1}{6}C_{IJK}p^Ip^Jp^K + \frac{1}{12}c_{2I}p^I\right)^{1/3}$. In writing the metric we have used the fact that AdS_3 can be written as S^1 fibered over AdS_2.

To proceed it is useful to introduce a new notation in which the auxiliary two-form field $v_{\mu\nu}$ can be treated as an additional gauge field in the theory with charge p^0. More explicitly we define $F^0_{\theta\phi} = \frac{p^0}{2}\sin\theta$ such that $v_{\theta\phi} = -\frac{3}{4}l_AF^0_{\theta\phi}$. Accordingly, we could introduce a new scalar field X^0 such that in the near horizon geometry one may set $X^0 = \frac{p^0}{l_A}$. Using this notation the constraint (4.7) can be written as

$$
\frac{1}{6}C_{IJK}X^IX^JX^K + \frac{1}{12}c_{2I}(X^0)^2X^I|_{p^0=1} = 1. \tag{4.12}
$$

Obviously working with this notation all expressions reduce to those we had for $p^0 = 1$, though it is not a solution for $p^0 \neq 1$. Nevertheless we will work with $p^0 \neq 1$ with the understanding that the solution is obtained by setting $p^0 = 1$. It is worth noting that in general the auxiliary field $v_{\mu\nu}$ cannot be treated as a gauge field, though it can be seen that for the models we are going to study it may be considered as a gauge field.

It is also useful to introduce indices $A, B, \cdots$ such that they take their values over 0 and $I, J, \cdots$ by which the equation (4.12) can be recast into the following form

$$
\frac{1}{6}C_{ABC}X^AX^BX^C = 1 \tag{4.13}
$$

where $C_{ABC} = C_{IJK}$ for $A, B, C = I, J, K$ and $C_{00I} = \frac{c_{2I}}{6}$ and the other components are zero. Thus, following the notion of very special geometry, it is natural to define X_A and the metric C_{AB} as follows

$$
X_A = \frac{1}{6}C_{ABC}X^BX^C, \qquad\qquad C_{AB} = \frac{1}{6}C_{ABC}X^C. \tag{4.14}
$$

It is easy to verify that

$$
X_AX^A = 1, \qquad X_A = C_{AB}X^B, \qquad C_{AB}X^AX^B = 1. \tag{4.15}
$$

In this language the magnetic central charge can also be defined as $Z_m = X_Ap^A$ and therefore the near horizon parameters can be fixed by extremizing it, i.e. $\partial_iZ_m = \partial_iX_Ap^A = 0$. Using (4.15) a solution would be $X^A = p^A/Z_m$. Plugging this into (4.13) we find

$$
Z_m = \left(\frac{1}{6}C_{ABC}p^Ap^Bp^C\right)^{1/3} = \left(\frac{1}{6}C_{IJK}p^Ip^Jp^K + \frac{(p^0)^2}{12}c_{2I}p^I\right)^{1/3}. \tag{4.16}
$$

Of course to get the final result one needs to set $p^0 = 1$ at the end of the computations.

By making use of this notation, with the understanding of $p^0 = 1$, the solution (4.11) can be written as follows

$$\begin{aligned} ds^2 &= \frac{Z_m^2}{4}\left(dx^2 - 2rdxdt + \frac{dr^2}{r^2} + d\theta^2 + \sin^2\theta\, d\phi^2\right), \qquad & X^A &= \frac{p^A}{Z_m}, \\ A_\theta^A &= -\frac{p^A}{2}\cos\theta\, d\phi, & Z_m^3 &= \frac{1}{6}C_{ABC}p^Ap^Bp^C. \end{aligned} \tag{4.17}$$

Following [10] we will consider the total space of U(1) bundle over (4.17) to define a six dimensional manifold with the metric

$$ds_6^2 = \frac{Z_m^2}{4}\left(dx^2 - 2rdxdt + \frac{dr^2}{r^2} + \sigma_1^2 + \sigma_2^2\right) + (2X_AX_B - C_{AB})(dy^A + A^A)(dy^B + A^B). \tag{4.18}$$

Here σ_i are right invariant one-forms such that $\sigma_1^2 + \sigma_2^2 = d\theta + \sin^2\theta\, d\phi^2$. Let us define new coordinates z, ψ through the following expressions

$$y^A = z^A + (\sin B - 1)X^AX_Bz^B - \frac{1}{2}p^A\psi, \qquad x = \frac{2\cos B}{Z_m}X_Az^A \tag{4.19}$$

where $\sin B$ is a constant which its physical meaning will become clear later. We define the coordinates such that the new coordinates have the following identification

$$\psi \sim \psi + 4\pi m, \qquad z^A \sim z^A + 2\pi n^A, \tag{4.20}$$

where m and n^A are integers. Accordingly one can read the identifications of y and x. Using the new coordinate ψ the right invariant one-forms can be defined by

$$\begin{aligned} \sigma_1 &= -\sin\psi d\theta + \cos\psi\sin\theta d\phi, \\ \sigma_2 &= \cos\psi d\theta + \sin\psi\sin\theta d\phi, \\ \sigma_3 &= d\psi + \cos\theta d\phi. \end{aligned} \tag{4.21}$$

In terms of the new coordinates the six dimensional metric (4.18) reads

$$\begin{aligned} ds^2 &= \frac{Z^2}{4}\left[-(\cos B\, rdt + \sin B\sigma_3)^2 + \frac{dr^2}{r^2} + \sigma_1^2 + \sigma_2^2 + \sigma_3^2\right] \\ &\quad + (2X_AX_B - C_{AB})(dz^A + \tilde{A}^A)(dz^B + \tilde{A}^B), \end{aligned} \tag{4.22}$$

where $\tilde{A}^A = -\frac{p^A}{2}(\cos B\ rdt + \sin B\sigma_3)$. The obtained six dimensional manifold can be treated as the total space of a U(1) bundle over BMPV black hole at R^2 level. Therefore we can reduce the metric (4.22) to five dimensions to get BMPV black hole where higher derivative corrections are also taken into account. The resulting five dimensional black hole solution is

$$\begin{aligned} ds^2 &= \frac{l_A^2}{4}\left[-(\cos B\, rdt + \sin B\sigma_3)^2 + \frac{dr^2}{r^2} + \sigma_1^2 + \sigma_2^2 + \sigma_3^2\right], \qquad X^I = \frac{p^I}{l_A}, \\ \tilde{A}^I &= -\frac{p^I}{2}(\cos B\, rdt + \sin B\sigma_3), \qquad \tilde{A}^0 = -\frac{1}{2}(\cos B\, rdt + \sin B\sigma_3), \end{aligned} \tag{4.23}$$

and the auxiliary field is given by $v = -\frac{3}{4} l_A \tilde{F}^0$. From this solution we can identify $\sin B$ as the angular momentum, J, of the BMPV solution through the relation $\sin B = \frac{J}{Z^3}$. As a result we have demonstrated that the total bundle space of near horizon wrapped M2's and wrapped M5's are equivalent up to R^2 level, generalizing the tree level results of [10]. In fact one may go further to show that both of them can be obtained from quotients of $AdS_3 \times S^3$ with a flat $\mathrm{U}(1)^{N-1}$ bundle, similar to tree level case considered in [10]. It is then possible to use this connection to increase our understanding of microstate counting of 5D supersymmetric rotating black hole arising from wrapped M2-branes in Calabi-Yau compactification of M-theory. We hope to come back to this point in our future publication.

5. Discussions and conclusions

In this paper we have studied higher order corrections to the entropy of extremal BPS black holes in five dimensions in which the higher order corrections come from supersymmetric completion of the Chern-Simons term. The explicit corrections have been written for a specific model, namely, STU model. To extend these results to a generic case we will have to solve a set of equations which can schematically be written as follows

$$C_{IJK} x^J x^K = a_I, \qquad I = 1, \cdots, N, \tag{5.1}$$

where C_{IJK} and a_I and given parameters. In general it is difficult to solve these equations and they may not even have a unique solution. Nevertheless, for a particular model they can be solved explicitly (for example see [24]). We have observed that the solution of the above equations can be used for both tree level case and the case when the R^2 corrections are included. The only difference is that in the presence of R^2 terms one just needs to replace a_I by another constant which is related by a constant shift.

In fact as far as the entropy is concerned we have seen that the corrected entropy can simply be obtained by replacing the electric charges q_I's with the shifted charges q_I^+'s defined in footnote 4. In particular this result shows how the higher order terms stretch the horizon leading to non-zero entropy out of a small black hole which in the tree level is singular with vanishing entropy.

We note that when the near horizon geometry is $AdS_3 \times S^2$ it has been shown in [6, 27] that the four derivative action gives the exact expression for the entropy in the large momentum limit. Following our study in the present paper, it is interesting to see if this is also the case when the near horizon geometry is $AdS_2 \times S^3$.

As an aside let us define C^{IJK} in terms of C_{IJK} by $C^{IJK} C_{KLM} = \delta^I_L \delta^J_M + \delta^I_M \delta^J_L$; although it is not clear whether this equation can be solved or even if it has a unique solution, it can be used to write a solution for equations (5.1). For example in terms of the electric charges the corresponding black hole entropy could be written as [25]

$$S = 2\pi \sqrt{\frac{1}{6} C^{IJK} q_I q_J q_K}. \tag{5.2}$$

In the presence of R^2 corrections the entropy has the same form, except that one needs to replace q_I by q_I^+.

Unfortunately the microscopic origin of the entropy of the five dimensional black hole we have been considering has not been fully understood (see however [9]). Therefore a priori it is not clear with what result it should be compared. Nevertheless we may compare the results to those obtained from another method. In particular one can use the 4D/5D connection to study the five dimensional black hole using the known four dimensional results [17]. We note, however, that to make this comparison more precise we need to understand the 4D/5D connection better specially when the supersymmetrized R^2 corrections are added [26].

We have also studied the black string solution in the presence of higher derivative terms. Using the corrected solution we have generalized the consideration of [10]. In particular we have shown that the total bundle space of near horizon wrapped M2's and wrapped M5's are equivalent up to R^2 level, generalizing the tree level results of [10]. Actually we could also show that both of them can be obtained from quotients of $AdS_3 \times S^3$ with a flat $\mathrm{U}(1)^{N-1}$ bundle, similar to tree level case considered in [10]. Although we have not pushed this observation any further, one might suspect that this connection could lead to better understanding of the microstate counting of 5D supersymmetric black holes arising from wrapped M2-branes in Calabi-Yau compactification of M-theory.

Note added: while we were in the final stage of the project the paper [28] appeared in the arXive where the five dimensional black hole in the presence of higher derivative terms given by (2.6) has been studied.

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